



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

## NOTE ON CAUCHY'S NUMBERS.

By PROF. A. S. CHESIN, Baltimore, Md.

Let  $p$  be any integer *positive, negative, or zero*;  $q$  and  $j$  any *positive integers or zero*; develop the expression

$$x^{-p} \left[ x + \frac{1}{x} \right]^j \left[ x - \frac{1}{x} \right]^q \quad (\text{A})$$

in powers of  $x$  and denote the constant term of this development by  $N_{-p,j,q}$ . The number  $N_{-p,j,q}$  is called a *number of Cauchy*. These numbers play an important role in certain developments used in Celestial Mechanics. In this note a formula will be given by which Cauchy's numbers can be directly and easily calculated.

First, let us recall some important properties of these numbers; namely, that

$$N_{-p,j,q} = 1, \quad (1)$$

when  $-p + j + q = 0$ ;

$$N_{-p,j,q} = 0, \quad (2)$$

when  $-p + j + q$  is a negative number or when it is odd;

$$N_{p,j,q} = (-1)^q N_{-p,j,q}; \quad (3)$$

$$N_{-p,j+1,q} = N_{-p+1,j,q} + N_{-p-1,j,q}; \quad (4)$$

$$N_{-p,j,q+1} = N_{-p+1,j,q} - N_{-p-1,j,q}. \quad (5)$$

From the last two formulas, by a successive application of the same, the following are easily obtained:

$$\begin{aligned} N_{-p,j,q} = & N_{-p+m,j-m,q} + \left[ \frac{m}{1} \right] N_{-p+m-2,j-m,q} + \left[ \frac{m}{2} \right] N_{-p+m-4,j-m,q} \\ & + \left[ \frac{m}{3} \right] N_{-p+m-6,j-m,q} + \dots + \left[ \frac{m}{1} \right] N_{-p-m+2,j-m,q} + N_{-p-m,j-m,q}, \end{aligned} \quad (6)$$

$$\begin{aligned} N_{-p,j,q} = & N_{-p+m,j,q-m} - \left[ \frac{m}{1} \right] N_{-p+m-2,j,q-m} + \left[ \frac{m}{2} \right] N_{-p+m-4,j,q-m} \\ & - \left[ \frac{m}{3} \right] N_{-p+m-6,j,q-m} + \dots + (-1)^m N_{-p-m,j,q-m}. \end{aligned} \quad (7)$$

In particular, if  $m = j$ , formula (6) gives

$$\begin{aligned} N_{-p,j,q} = N_{-p+j,0,q} + \left[ \frac{j}{1} \right] N_{-p+j-2,0,q} + \left[ \frac{j}{2} \right] N_{-p+j-4,0,q} \\ + \left[ \frac{j}{3} \right] N_{-p+j-6,0,q} + \dots + N_{-p-j,0,q}; \end{aligned} \quad (6')$$

and if  $m = q$ , formula (7) gives

$$\begin{aligned} N_{-p,j,q} = N_{-p+q,j,0} - \left[ \frac{q}{1} \right] N_{-p+q-2,j,0} + \left[ \frac{q}{2} \right] N_{-p+q-4,j,0} \\ - \left[ \frac{q}{3} \right] N_{-p+q-6,j,0} + \dots + (-1)^q N_{-p-q,j,0}. \end{aligned} \quad (7')$$

Combining formulas (6') and (7') together, we easily obtain the following :

$$\begin{aligned} N_{-p,j,q} = N_{-p+j+q,0,0} + (j-q) N_{-p+j+q-2,0,0} \\ + \left[ \left[ \frac{j}{2} \right] - \left[ \frac{j}{1} \right] \left[ \frac{q}{1} \right] + \left[ \frac{q}{2} \right] \right] N_{-p+j+q-4,0,0} + \dots \\ + \left[ \left[ \frac{j}{n} \right] - \left[ \frac{j}{n-1} \right] \left[ \frac{q}{1} \right] + \left[ \frac{j}{n-2} \right] \left[ \frac{q}{2} \right] - \dots + (-1)^n \left[ \frac{q}{n} \right] \right] N_{-p+j+q-2n,0,0} \\ + \dots + (-1)^q N_{-p-j-q,0,0}. \end{aligned} \quad (8)$$

But

$$N_{-p+j+q-2n,0,0} = 0, \text{ if } -p+j+q-2n \geq 0,$$

$$N_{-p+j+q-2n,0,0} = 1, \text{ if } -p+j+q-2n = 0.$$

In fact, if  $j = q = 0$  in formula (A), the constant part in the development is zero, unless  $p = 0$ , in which case it is unity. Hence

$$\begin{aligned} N_{-p,j,q} = \left[ \frac{j}{n} \right] - \left[ \frac{j}{n-1} \right] \left[ \frac{q}{1} \right] + \left[ \frac{j}{n-2} \right] \left[ \frac{q}{2} \right] - \dots + (-1)^n \left[ \frac{q}{n} \right] \\ n = \frac{1}{2}(-p+j+q). \end{aligned}$$

This formula gives the solution of the proposed problem.